CONVECTIVE MOTION OF A GAS RESULTING FROM THE PROPAGATION OF A HEAT WAVE ALONG THE LOWER BOUNDARY OF A CLOSED REGION

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Many natural and engineering processes occur under conditions of convection. The structure of the convective motion that arises depends on the past history of the process and on how the convection is induced. We study the convective motion of a gas in a two-dimensional square cross-section of space when a temperature jump (thermal wave) propagates along the lower boundary. It is shown that beyond the stability threshold of the system to symmetric and asymmetric perturbations, depending on the wave propagation velocity, two stationary solutions are possible which correspond to single-vortex and two-vortex convection structures. Using the physics of the problem, we obtain an approximate expression for the boundary separating the domains of the two stationary solutions. In order to specify the conditions under which the different convective motions are formed, we carry out a numerical simulation of the development of convection using the nonstationary two-dimensional Navier—Stokes equations for a compressible gas. The time and energy characteristics of both types of convective motion are established. We consider an example of our model in which a temperature jump moving with a constant velocity can be considered as propagating along the surface of a catalyst or as a fuel wave in an exothermal reaction in order to describe the gasdynamics in combustion in a closed volume.

1. Consider a gas at rest confined to a square $0 \leqslant X, Y \leqslant L$, where X and Y are Cartesian coordinates. Let the gas be in equilibrium in an external force field (gravity) at temperature T_0. The external force is directed along the negative Y direction. On the horizontal boundaries of the region the temperature is constant and equal to T_0, and the lateral (vertical) boundaries are thermally insulated.

At $t = 0$ a temperature jump begins to propagate with constant velocity w along the lower boundary from the left vertical wall toward the right. After the jump has passed a certain point, the temperature is raised to the value $T = T_S$. The wave propagation is described by the nonstationary boundary condition

$$Y = 0,\ T/T_0 = 1 + (\theta_s - 1)\Theta(wt - X), \tag{1.1}$$

where is the Heaviside function and $\theta_S = T_S/T_0 > 1$. In the time interval $0 < t < t_B = L/w$ the boundary condition (1.1) leads to an asymmetric perturbation in the initially at rest gas. Behind the temperature jump, heat waves propagate in the fluid layers adjacent to the lower boundary. After the temperature jump reaches the right boundary ($t \geqslant t_B$) the boundary condition (1.1) specifies a constant temperature on the entire lower boundary and now acts symmetrically on the system. The effect of symmetric or asymmetric perturbations in the development of convection will depend on the transit time of the wave t_B. For small t_B symmetric perturbations are most significant; for large t_B asymmetric perturbations are most significant.

These perturbations lead the system away from the equilibrium state and they can induce various types of convective motion. It was shown in [1] that in a plane square region a symmetric perturbation (a suddenly applied external force acting along the Y axis; heating and cooling of the lower wall) induces stationary two-vortex convective motion. An asymmetric perturbation (rotation of the external force vector) produces single-vortex motion. Thus when t_B is small (the thermal wave velocity w is large) one expects the formation of two vortices symmetric about the axis $X = 0.5L$ and for large t_B (small velocity) a single vortex with a rising flow of gas near the left boundary.

We discuss the conditions under which the two convective structures are realized. It was shown in [2, 3] that convection develops after a characteristic time, called the

Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 44-50, July-August, 1983. Original article submitted June 28, 1982.

convection induction period. During the induction period there is a heating of the gas near the heated wall by conduction and under the action of gravity an upward convective flow is formed. The realization of a certain type of convective motion under the heating condition (1.1) depends on the relation between the propagation time of the wave t_B and the convection induction period. If t_B is larger than the induction time for single vortex convective motion t_1, then until the wave reaches the right boundary, single-vortex convection will arise near the left boundary. If on the other hand $t_B < t_1$, then convection only begins to develop after the temperature T_s is established on the entire lower boundary and therefore under the action of a symmetric perturbation. Hence after the induction time t_2 there will be two symmetric vortices.

We estimate the critical velocity of the thermal wave which gives the boundary between the two branches of the solution. Let t_* be the smallest wave transit time for which single-vortex motion can occur. After this time, near the left boundary of the region where single-vortex convection has developed, a heated layer of thickness $l \sim 2\sqrt{\varkappa t_*}$ will be formed where $\varkappa$ is the thermal conductivity. The velocity of a buoyant mass of gas heated to temperature T_s of characteristic linear dimension l is of order $V_c^2 \sim lg(\theta_s - 1)/\theta_s$ [4] where g is the acceleration of gravity. The characteristic time for the development of convection during which the heated gas acquires velocity V_c can be estimated as

$$t_1 \sim \frac{l}{V_c} = \left[\frac{l}{g(\theta_s - 1)}\right]^{1/2}. \tag{1.2}$$

The condition $t_1 \approx t_*$ defines the boundary between the domains of the symmetric and asymmetric branches of the solution; from (1.2) we have

$$t_B = t_* = \left(\frac{2\varkappa}{g(\theta_s - 1)}\right)^{2/3} = \frac{L^2}{\varkappa}\frac{1}{k(\mathrm{Ra\,Pr})^{2/3}}$$
$$(\mathrm{Ra} = L^3 g(\theta_s - 1)/\nu\varkappa,\ \mathrm{Pr} = \nu/\varkappa) \tag{1.3}$$

or

$$u = u_* = k(\mathrm{Ra\,Pr})^{1/6}\,(u = w\theta_s^{2/3}/(Lg(\theta_s - 1))^{1/2}), \tag{1.4}$$

where u is the wave velocity in units of the characteristic convective velocity, Ra is the Rayleigh number, Pr is the Prandtl number, k is a constant of proportionality equal to 0.63 according to the above estimate.

If $u < u_*(t_B > t_*)$, stationary single-vortex motion is realized. The minimum linear dimension of the heated layer near the left wall for this motion to occur is $l_*^2 = 4\varkappa t_B = 4L^2/k(\mathrm{RaPr})^{2/3}$. For a given wave transit time t_B the induction period for single-vortex convective motion is between t_* and t_B (i.e., $t_* < t_1 < t_B$). If $u > u_*(t_B < t_*)$, then the time-dependent boundary condition (1.1) does not affect the development of convection and two-vortex motion is formed.

The above discussion makes sense only for Rayleigh numbers exceeding a critical value Ra_2 which corresponds to the loss of stability with respect to symmetric perturbations [1, 5]. For $\mathrm{Ra} < \mathrm{Ra}_2$ only single-vortex motion can occur and for $\mathrm{Ra} > \mathrm{Ra}_2$ both types of stationary motion are possible depending on whether the wave velocity is larger or smaller than the critical value $u = u_*(\mathrm{Ra}, \mathrm{Pr})$.

2. In order to make the estimates (1.3) and (1.4) quantitative, and to establish the stationary characteristics of convection, we study numerically the development of convection in a gas confined to a plane square region with the propagation of a temperature jump along the lower boundary. The problem is formulated in dimensionless form:

$$\frac{\partial \mathbf{U}}{\partial \tau} + (\mathbf{U}\nabla)\mathbf{U} = -\frac{1}{\gamma\rho M^2}\nabla p + \mathbf{j} + \frac{1}{\rho\,\mathrm{Re}}\left(\Delta\mathbf{U} + \frac{1}{3}\operatorname{grad}\operatorname{div}\mathbf{U}\right), \tag{2.1}$$
$$\frac{\partial \rho}{\partial \tau} + \operatorname{div}\rho\mathbf{U} = 0,\ \frac{\partial\theta}{\partial\tau} + \mathbf{U}\nabla\theta = \frac{\gamma}{\rho\,\mathrm{Re\,Pr}}\Delta\theta - (\gamma - 1)\theta\operatorname{div}\mathbf{U},$$
$$p = \rho\theta(\tau = t(g/L)^{1/2},\ x = X/L,\ y = Y/L,\ \Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2,$$
$$\mathbf{j} = (0, -1),\ M^2 = Lg(\gamma RT_0)^{-1},\ \mathrm{Re} = L^{3/2}g^{1/2}\rho_0/\eta,\ \mathrm{Pr} = \lambda/c_p\eta);$$
$$\tau = 0,\ \mathbf{U} = 0,\ \theta = 1,\ p = \exp(-\gamma M^2 y); \tag{2.3}$$
$$\mathbf{U}|_\Gamma = 0,\ \theta'_x(x = 0;\ 1) = 0,\ \theta(y = 1) = 1; \tag{2.3}$$
$$\theta(y = 0) = 1 + (\theta_s - 1)\Theta(w'\tau - x)\ (w' = w/(Lg)^{1/2}). \tag{2.4}$$

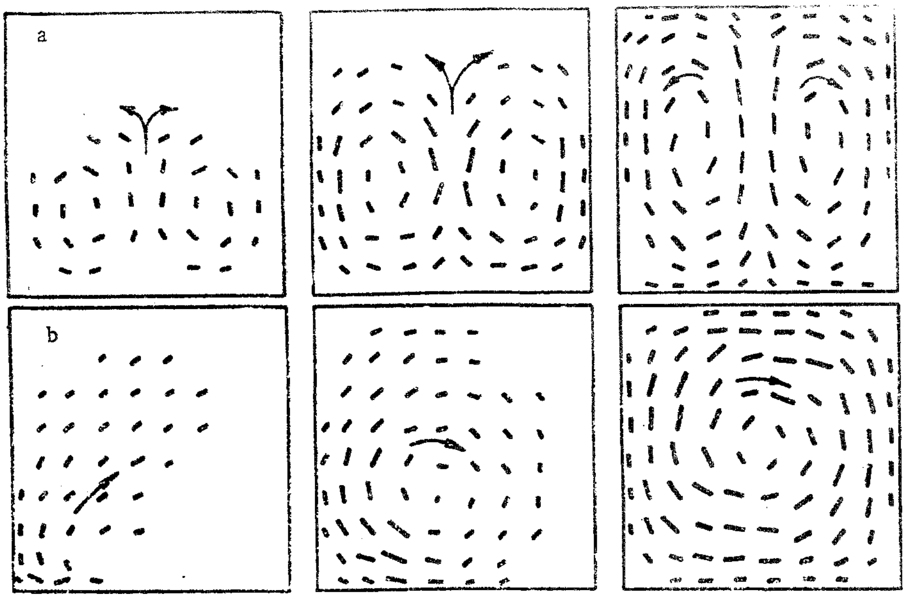

Fig. 1

Here we choose the following scales of measurement: length L, time $(L/g)^{1/2}$, velocity $(Lg)^{1/2}$, initial density ρ_0 at $y=0$, temperature T_0, pressure $\rho_0 RT_0$ (R is the gas constant), U is the velocity of the gas, ρ, p, and θ are the density, pressure, and temperature, $\gamma = c_p/c_V$ is the adiabatic index, M, Re, Pr are the Mach, Reynolds, and Prandtl numbers, and Γ is the boundary. The coefficients of thermal conductivity λ and dynamical compressibility η are taken to be constant, and viscous dissipation in the heat equation is neglected.

The system of equations (2.1) with the initial and boundary conditions (2.2) and (2.3), (2.4) is numerically integrated [6, 7] with the use of a rectangular 21×21 grid compressed near the boundary. In particular, near the boundary the grid points have twice the density as in the center of the region. The time step is chosen from the condition $\Delta\tau = KMh$, where the number K ranges from 2 to 8, and h is the minimum grid step. Control calculations were performed on a 41×41 grid with a different distribution of grid points. The mass and energy balances were satisfied to within 1% and 2% respectively. Five minutes of computer time was required to calculate 100 time steps.

The following values of the parameters were used in the calculations: $M^2 = 0.05$, $Pr = 1$, $\gamma = 1.4$, $w' = 1$–5, $Re = 130$–800, $\theta_s = 1.5$–4. The Rayleigh number was varied in the range $Ra = 9\cdot 10^3$–$5\cdot 10^5$. It should be pointed out that for the above values of θ_s one must take into account the compressibility of the medium in (2.1).

The results of the calculations support the existence of symmetric and asymmetric convective structures (Fig. 1, $Re = 200$, $\theta_s = 1.5$, $U = 0.6$). For large wave velocities stationary motion is established with two vortices having upward motion in the center and downward motion at the periphery (Fig. 1a with $u = 3.6 > u_*$ shows the velocity field at times $t = 8.3$, 16.7, 83.5). For small wave velocities, single-vortex stationary motion is formed with an upward motion of gas near the left boundary and downward motion along the right boundary (Fig. 1b, with $u = 3.15 < u_*$, $t = 5.6$, 11.2, 56.0).

The results of the calculations were used to construct the domains for the two stationary solutions on the (u, lg Ra) plane. In Fig. 2 the points labeled 1 correspond to steady-state single-vortex motion, and the points 2 to two-vortex motion. Only those points close to the boundary dividing the convective regimes are shown. The quantitative equation for the boundary curve for $Ra > 31{,}500$ has the form (1.3), (1.4), as before, but now $k = 0.68$ (this is shown by the solid straight line in Fig. 2).

The dependence $u_* \sim Ra^{1/6}$ is violated for small Rayleigh numbers because of the approach to the critical value Ra_2 corresponding to loss of stability with respect to symmetric perturbations. It was shown by calculation that $Ra_2 = 8600 \pm 100$. This agrees with the numerical result 8500 ± 200 done in [8] and the result from small perturbation stability analysis (8495) carried out in [5].

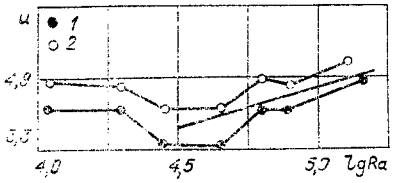

Fig. 2

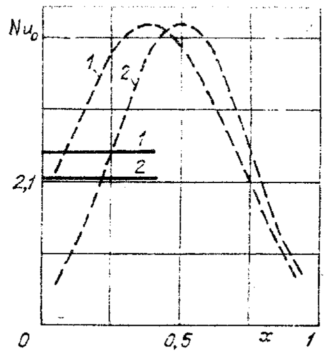

Fig. 3

Calculations for Ra $<Ra_2$ show that only single-vortex motion occurs in this range independent of the wave velocity. In the range Ra $>Ra_2$ near the branching boundary of the solutions (1.3), (1.4) from the direction of velocities $u<u_*$, an interaction of perturbations of different symmetry in the nonstationary part of the process is observed. Initially two vortices are formed which then deteriorate into a single vortex with positive rotation.

The induction time for two-vortex convection is

$$t_2 = t_2^0 + t_B \quad (t_B < t_*),$$

where t_2^0 is the induction period for $t_B \to 0$, and is determined by the numerical solution of (2.1) to (2.3) with the condition $\theta(y=0) = \theta_s$ on the lower boundary of the region for the time corresponding to the maximum in the total kinetic energy. From analysis of the calculated data we obtain the formula

$$t_2^0 = \frac{L^2}{\varkappa}\frac{38.22}{(Ra - 8000)^{0.638}P_r^{1/6}}.$$

The induction period for two-vortex convection for finite wave velocity satisfies the inequality $t_2^0 \leqslant t_2 \leqslant t_2^0 + t_B$ $(t_B < t_*)$ and can be larger or smaller than the induction period for single-vortex convection $t_* < t_1 < t_B$ $(t_B > t_*)$.

Starting from Ra$\approx 8.5\cdot 10^4$ symmetric two-vortex motion is metastable for $u > u_*$. After the establishment of two-vortex motion where vibrations are damped out, vibration of all quantities begins again and the system transforms into the single-vortex state. The metastable state exists over a long time interval which exceeds the propagation time of the thermal wave along the lower boundary by at least an order of magnitude; therefore one can speak of a quasistationary state for Ra $>8.5\cdot 10^4$. The existence of similar metastable states in an incompressible fluid was pointed out in [8], where detailed numerical studies were done on their origin and decay with the help of finite-difference methods.

3. The numerical solution of (2.1) to (2.4) up to the onset of convection gives the stationary characteristics of the convection such as the mean heat fluxes at the lower and upper boundaries:

$$Nu_s = \int_0^1 q(x, 0)\,dx, \quad Nu_0 = \int_0^1 q(x, 1)\,dx \quad (q = -\theta_y'/\theta_s),$$

and the kinetic and thermal (internal) energies of the entire mass of gas in the region:

$$E = \frac{gL^3}{\nu^2}\int_0^1\int_0^1 \frac{1}{2}\rho U^2 dxdy, \quad H = \frac{c_V T_0 L^2}{\nu^2}\int_0^1\int_0^1 \rho\theta dxdy.$$

Here the heat fluxes are normalized by their stationary values in the absence of convection and the energies are normalized by $\rho_0\nu^2$. In the stationary regime $Nu_s = Nu_0$.

The results of the calculations can be summarized as follows ($P_r = 1$). For single-vortex motion

$$Nu_1 = 1.79r_1 - 5.16\ (Ra > 7400,\ r_1 = \lg Ra); \tag{3.1}$$

$$\lg E_1 = 1.29r_1 - 3.31,\ \lg H_1 = r_1 + 1.24\ (3.93 < r_1 < 5.4); \tag{3.2}$$

and for two-vortex motion

$$\mathrm{Nu}_2 = 1 + 0.01(\mathrm{Ra} - 8600)^{0.5}(\mathrm{Ra} < 4.9 \cdot 10^4),\ \mathrm{Nu}_2 = 3\,(\mathrm{Ra} > 5 \cdot 10^4); \tag{3.3}$$

$$\lg E_2 = 1.05 r_2 - 2.33,\ \lg H_2 = 3.5 + 0.5 r_2 (3.41 < r_2 = \lg(\mathrm{Ra} - 8600) < 5.38). \tag{3.4}$$

Formulas (3.1) and (3.3) approximate the numerical results within 5% and in the indicated ranges of Rayleigh numbers agree with numerical simulations of convection obtained earlier for an incompressible liquid [5, 8, 9] (in these papers it was assumed that a linear temperature dependence obtained on the vertical walls, unlike the present paper) and in gases [1]. In particular, near the stability threshold the following relation is satisfied very accurately $(\mathrm{Nu}_2 - 1)^2 \sim (\mathrm{Ra} - \mathrm{Ra}_2)$.

Equations (3.2) and (3.4) determine E and H to 20% accuracy. For Rayleigh numbers which only slightly exceed the critical values Ra_2, the kinetic energy of a gas is much larger for single-vortex convection than for two-vortex motion. As Ra increases, i.e., as we go away from the stability boundary for symmetric perturbations, the vortex energy increases and the difference between E_1 and E_2 rapidly decreases. In both convective regimes the kinetic energy of a gas is much less (by about a factor of 10^3 to 10^4) than the thermal energy.

4. Problems where convection is induced by a moving heat source are of interest in diverse applications. For example, such a mechanism has been used to explain the circulation in the atmosphere of Venus [10, 11]. We consider here two examples which illustrate the model and numerical results, as applied to motion arising from an exothermal chemical reaction in a closed vessel.

Many exothermal chemical reactions proceed on catalytic surfaces which form the boundaries of the reactor. If the stationary state is not unique for such a heterogeneous catalytic system, local perturbations on the catalyst surface can lead under certain conditions to the propagation of a traveling wave of the other stationary state along the surface [12, 13]. For example in the oxidation of sulfur-dioxide on a platinum catalyst, the combustion causes a transition from the low-temperature kinetic regime of the reaction to the diffusive regime. Numerical calculations and experiment [13] both show that a combustion wave propagates from the local combustion center with an approximately constant velocity behind the front which establishes the high temperature. The wave, which can be modeled as a moving temperature jump, has a velocity which depends on the reaction kinetics, the state of the catalyst, the thermal and diffusive regimes of the catalytic surface and other factors, and can vary widely. The solution obtained above can be used to establish the nature of the motion of the reacting mixture resulting from the firing of the catalyst. In a closed reactor symmetric and asymmetric vortex motions will arise, and these lead to different distributions of temperature and concentrations of the reacting materials over the volume of the reactor. The realization of either state depends on the wave velocity (or the transit time of the wave) and is given by (1.3), (1.4).

The other example involves the propagation of a combustion wave along a planar surface of a combustible material. If the chemical transformation region above the surface of the combustible material is much smaller than the spatial scale of the resulting gasdynamic structures, then the reaction can be considered as taking place on the surface itself, and the propagation of the combustion wave will approximately be described by (1.1). Then if the closed region is large enough, the solution of (2.1) to (2.4) will describe the gasdynamics from the propagation of the combustion wave along the lower boundary of the enclosure whose length is assumed to be much less than that of the other horizontal dimension of the enclosure (the corridor fire model). Depending on the fire propagation velocity, the gas undergoes symmetric or asymmetric convective motion and consequently different thermal loads will be realized on the walls of the enclosure. The magnitudes of these loads are basic for determining the fire safety of the enclosure. We illustrate the situation in Fig. 3 where for $\mathrm{Ra} = 2 \cdot 10^4$ is shown the mean Nu (straight lines) and local $q(x, 1)$ (dashed curves) steady-state heat fluxes on the upper boundary of the region for single-vortex (curves 1) and two-vortex (curves 2) motion. The mean heat flux for single-vortex convection is about 1.2 times larger than for symmetric motion. The maximum heat flux in the first case is shifted toward the left wall ($x \approx 0.38$); naturally for symmetric motion the maximum lies on the symmetry axis. The maximum heat fluxes are about equal for both cases and significantly exceed their average values. Therefore in estimating the heat-stability it is important to use the maximum value of the local heat flux and also its location.

Finally when the planar region is not a square [14, 15], (1.4) gives the wave velocity dividing the symmetric and asymmetric cases; in this case for L one takes the length of the boundary along which the thermal wave propagates.

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